MOONS - Derivation of Conservative FD Formulation

## Dimensional induction equation

Maxwell's equations (while neglecting the displacement current):

Solving for the electric field in terms of the current in Ohm's law yields

Plugging this into Faraday's Law yields

Distributing we have

Applying Ampere's Law to the current yields

Using the vector identity

And noting that

We can write our advective term as

Moving everything to the RHS, we may write our equation in vector form as

Or, more explicitly, in mixed vector-index notation

Using the kronecker delta identity, , the last term is

So we may write

So we finally have a dimensional *conservative* Finite Difference Method (FDM) formulation that is prepared to be integrated for a Finite Volume Method (FVM) formulation:

## Non-dimensionalizing

Introducing the following reference values to non-dimensionalize by

Substituting the magnetic field and canceling the reference factor we have

Now substituting the velocity and time we have

Multiplying out we have

Removing the asterisks we have

Introducing the characteristic magnetic Reynolds number

## Expanded form

Expanding the dummy index gives us the equation for the ith component:

## Expanded form for uniform properties

Expanding the dummy index gives us the equation for the ith component:

Or